DOCUMENT RESUME

ED 205 405 SE 035 871

AUTHOR Edlund, Milton C.
TITLE Introduction to Reactor Statics Modules, RS-1. Nuclear Engineering Computer Modules.
INSTITUTION Virginia Polytechnic Inst. and State Univ., Blacksburg. Coll. of Engineering.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE [71]
GRANT NSF-GZ-2888
NOTE 25p.

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS College Science; Computer Oriented Programs; Engineering; *Engineering Education; Engineering Technology; Higher Education; *Instructional Materials; *Mathematical Models; *Nuclear Physics; Physics; *Program Descriptions; Quantum Mechanics; Science Education

ABSTRACT

The nine Reactor Statics Modules are designed to introduce students to the use of numerical methods and digital computers for calculation of neutron flux distributions in space and energy which are needed to calculate criticality, power distribution, and fuel burn-up for both slow neutron and fast neutron fission reactors. The diffusion approximation is used for the calculation of neutron transport. Collision probabilities are used to calculate the effect of heterogeneous lattices of fuel and moderator on resonance absorption and slow neutron disadvantage factors. In this module, the three major problems of neutron diffusion in slow neutron fission reactors are reviewed. An energy-dependent diffusion equation, steady state epithermal multi-group equations, and a slow neutron diffusion equation are introduced. The structure and organization of the remaining eight modules are outlined. These modules are intended to supplement textbooks and other lecture material generally available to students in their course work. It is assumed that students are familiar with elementary nuclear structure, neutron-nuclei interactions, and introductory material on fission chain reactors. (Author)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED205405

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Patricia Babb of the NSF
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 035 471

INTRODUCTION TO REACTOR STATICS MODULES, RS-1

by

Milton C. Edlund

The University gratefully acknowledges the support of the Division of Higher Education of the National Science Foundation for support of this work performed under Grant GZ-2888 and the support of Duke Power Company, North Carolina Power and Light Company, and Virginia Electric and Power Company.

Project Director: Milton C. Edlund

TABLE OF CONTENTS

		Page
1.1	The Three Major Problems of Neutron Diffusion in Slow Neutron Fission Reactors	3
1.2	An Energy Dependent Diffusion Equation.	5
1.3	Steady State Epithermal Multi-Group Equations	9
1.4	Slow Neutron Diffusion Equation	16
1.5	Structure and Organization of the Reactor Statics Modules	17

INTRODUCTION TO REACTOR STATICS MODULES, RS-1

Preface

The development of reactor physics shortly after the discovery of nuclear fission in January 1939 is truly an elegant application of nuclear physics and applied mathematics. The notion of a nuclear chain reaction was intensively pursued after the discovery of fission.[1] Nuclear physicists at that time realized that neutrons would be emitted in the fission process. And since neutrons have no electrical charge, they would easily penetrate the electric field surrounding a nucleus to interact with it and thereby serve as the carrier for a chain reaction.

During the early months of 1940 it was shown that fission of uranium was primarily induced by slow neutron absorption in the less abundant uranium -235 isotope. With remarkable rapidity, the physical problems of constructing a self-sustaining chain reaction in natural uranium were identified and solved leading to the first nuclear fission chain reaction at the University of Chicago on December 12, 1942.

Reactor physics developed during the Manhattan Project[2] as an inspired combination of some elements of mathematical physics and the definition and measurement of certain nuclear cross-sections and integral quantities related to the mathematical models used in the design of the plutonium production reactors built at Hanford, Washington during World War II. The models and analytical methods, which can be identified as "Integral Reactor Theory," were devised to minimize computations which were performed on desk calculators. Integral reactor theory is described in several textbooks,[3,4,5,6] which have

been used in nuclear engineering education during the last two decades.

Two developments have led to a significant advance in the methods of reactor physics analysis. The first is the more accurate measurement of a large number of nuclear cross sections. The second is the development of high speed and large capacity digital computers. The latter permits the use of more sophisticated mathematical models for computation; the former provides the data required to make the more sophisticated models useful in the design of nuclear reactors. The use of digital computers also facilitates the exploration of many core design variables and permits the solution of many problems which simply could not be solved otherwise.

The Reactor Statics Modules are designed to introduce students to the use of numerical methods and digital computers for calculation of neutron flux distributions in space and energy which are needed to calculate criticality, power distribution and fuel burnup for both slow neutron and fast neutron fission reactors. The diffusion approximation is used for the calculation of neutron transport. Collision probabilities are used to calculate the effect of heterogeneous lattices of fuel and moderator on resonance absorption and slow neutron disadvantage factors.

These Modules are intended to supplement textbooks and other lecture material generally available to students in their course work. It is assumed that students are familiar with elementary nuclear structure, neutron-nuclei interactions and introductory material on fission chain reactors at a level of Nuclear Reactor Engineering, S. Glasstone and A. Sesonske, Van Nostrand Reinhold Company, New York, N.Y. (1967).

1.1 The Three Major Problems of Neutron Diffusion in Slow Neutron Fission Reactors

There are three problems in diffusion theory, the solutions of which are necessary for the calculation of the multiplication factor in slow neutron reactors. They are,

1) the slowing down of fission neutrons to thermal energies without capture,
2) thermalization and diffusion of slow neutrons and
3) the slowing down of fission neutrons taking into account the possibility of their capture in the resonance region.

The solution of these problems provides the neutron distribution in energy and time throughout the reactor. The neutron distribution and the macroscopic cross sections give neutron-nuclei reaction rates, which are needed to calculate the multiplication factor and power distributions. The neutron distribution is the primary dependent variable required for the nuclear design of reactors.

The slowing down of neutrons is due almost entirely to elastic scattering with moderator nuclei. In light nuclei the first excited states lie about 1 Mev above the ground state, and hence, inelastic collisions are relatively unimportant in the moderation process. Particularly, since nuclei which are good moderators by virtue of the elastic scattering mechanism are of necessity light elements. Inelastic scattering in the heavy elements can reduce the energy of fast neutrons only down to about 0.1 Mev, which is the excitation energy of the first excited state in heavy nuclei. This process is important, however, in determining the slowing down length in close-packed uranium oxide fueled light

water reactors. It is less important in gas cooled graphite moderated reactors and is a primary process in fast breeder reactors.

Every elastic collision of a neutron with a moderator nucleus will reduce the energy of the neutrons until the kinetic energy of the neutron approaches an energy on the order of the thermal energy of the moderator atoms. At this stage, energy may be transferred from the moderator atoms to the neutron and the neutron energy will approach "thermal energy" asymptotically.

Above about 1 ev the moderator atoms can be considered to be at rest compared to the motion of the neutrons and the calculation of the energy loss per scattering collision is an elementary problem in particle mechanics. Slowing down above 1 ev is discussed in Module RS-3.

The last phase of the slowing down process, however, is quite difficult to analyze. Strictly speaking, these are no thermal neutrons in the sense of neutrons being in thermal equilibrium with the moderator. Because of the 1/v law of neutron absorption, the absorption probability increases at the end of the slowing down process and neutrons are actually absorbed before attaining the energy distribution which corresponds to thermal equilibrium. The diffusion and energy distribution of slow neutrons (below ~1 ev) is further complicated by the effect of chemical binding and, in the case of crystal lattices such as graphite, the effect of coherent scattering of low energy neutrons by many nuclei in the crystal. This problem is discussed in Module RS-5.

The calculation of the capture of neutrons in the resonances of nuclei is complicated by the rapid variation of cross sections with energy. Nevertheless, the theory of this process is quite satisfactory and calculations of resonance capture can be made accurately and quickly with digital computers. This problem is discussed in Module RS-4.

1.2 An Energy Dependent Diffusion Equation

Diffusion theory is widely used to calculate the neutron distributions needed for reactor design. The basic approximation of diffusion theory is Fick's law, which states that the neutron current per unit energy is proportional to the gradient of the neutron flux per unit energy; i.e.,

$$\underline{J}(\underline{r}, E, t) = - D(\underline{r}, E) \nabla \phi(\underline{r}, E, t). \qquad (1.2.1)$$

The diffusion coefficient is approximately given by,

$$D(\underline{r}, E) \tilde{=} \frac{1}{\Sigma_t(\underline{r}, E) - \sum_i \bar{\mu}_i(\underline{r}, E)\, \Sigma_{si}(\underline{r}, E)} \qquad (1.2.2)$$

Σ_t is the total macroscopic cross section, Σ_{si} is the scattering cross section and $\bar{\mu}_i$ is the average cosine of the scattering angle for the i th isotope. The summation is over all isotopes.

Both the current and the flux are distribution functions in space, $\underline{r}$, and energy, E. That is, the neutron current for neutrons in the volume element $d\underline{r}$ at $\underline{r}$ and having kinetic energies in the range E to E + dE at time, t, is,

$$\underline{J}(\underline{r}, E, t) d\underline{r}\, dE.$$

Similarly, $\phi(\underline{r}, E, t)\, d\underline{r}\, dE$ gives the number of neutrons in $d\underline{r}$ at $\underline{r}$ with energies in the range E to E + dE at time, t, multiplied by their speed, $v = \sqrt{2\, E/m}$.

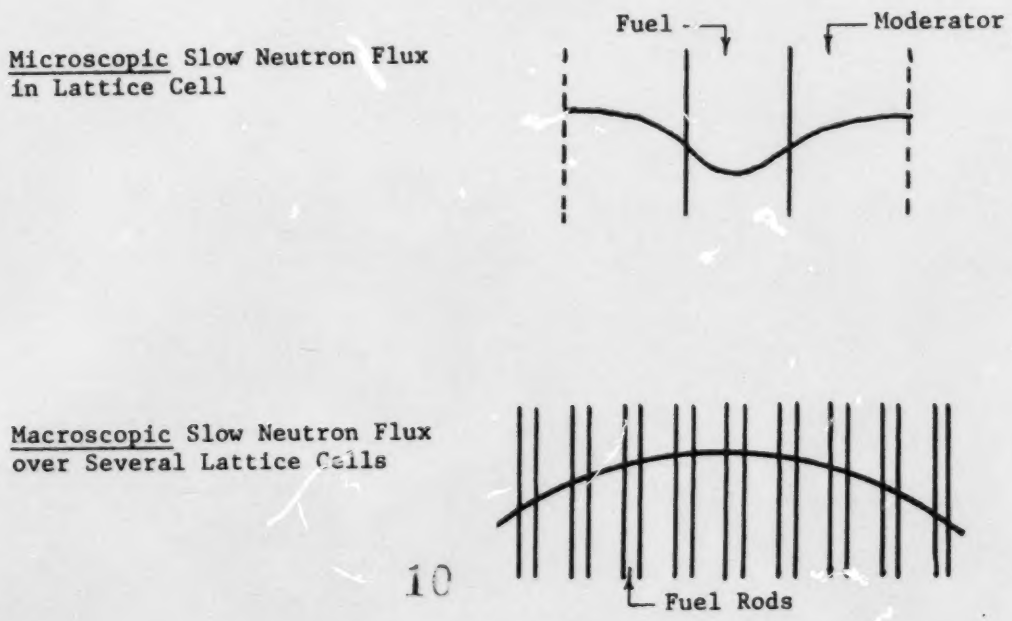
Fick's law and the formula for the diffusion coefficient can be derived as a first order approximation from the exact neutron transport theory.[7] The approximation is sufficiently accurate for those reactor design problems in which the variation of the flux with $\underline{r}$ is small compared to a neutron mean free

path and the variation of flux with energy at each position is small over the range of the average neutron energy change resulting from a scattering collision.

The diffusion approximation can be rationalized by noting that the simplest relation between the vector field, $\underline{J}$, and the scalar field, ϕ, which is invariant to rotation and translation of the coordinate system, is precisely Equation (1.2.1). After all, the neutrons in a reactor don't "know" what coordinate system is being used by the analyst!

Diffusion theory, however, is not sufficiently accurate for design requirements in the calculation of neutron distributions within a single lattice of fuel, clad and moderator. These distributions require more accurate neutron transport models. On the other hand, the migration of neutrons over a number of lattices can be treated by diffusion theory. The classical approach is to divide the neutron distributions into "macroscopic" and "microscopic" components as illustrated in Figure 1.2.1.

Figure 1.2.1



The microscopic distributions, those within a fuel lattice cell, are computed using the more accurate transport models to obtain cell average macroscopic cross sections. These cross sections are then used in diffusion equations to calculate the average neutron distributions over a number of lattices; i.e., the "macroscopic" distributions. An introduction to the calculation of the "microscopic" distributions is presented in Modules RS-4 and 6.

Returning to the "macroscopic" diffusion theory, the basic principle is simply neutron conservation in an element $d\underline{r}\ dE$ at $\underline{r}$ and E. To simplify the presentation, consider a single fissionable isotope such as uranium -235 mixed with a moderator consisting of a single isotope like graphite. Further, assume a dilute system in which the critical concentration of uranium -235 is so small that scattering by the uranium can be neglected. The most important neutron-nuclei interactions are fissions below about 1 ev and elastic scattering of neutrons by moderator nuclei. The time rate of change of the number of neutrons in $d\underline{r}\ dE$ is equal to the sum of

1) the rate at which neutrons are scattered into dE by moderator nuclei

and

2) the rate of production of fission neutrons

minus

3) the rate of leakage by diffusion

and

4) the rate of removal by all interactions with the uranium and moderator.

The number of scattering collisions in $d\underline{r}\ dE'$ at time, t, is

$$\Sigma_s(\underline{r},E')\ \phi\ (\underline{r},E',t)\ d\underline{r}\ dE'$$

$\Sigma_s(\underline{r},E')$ is the macroscopic scattering cross section of moderator. Let

$$P(E' \to E)dE$$

be the probability that neutrons scattered with initial energy, E', have final energy in the range E to E + dE. The total number of neutrons scattered into dE is obtained by integration with E' from 0 to 10 Mev and gives the first term in the neutron balance, which is

$$d\underline{r}\ dE \int_0^{10\ \text{Mev}} \Sigma_s(\underline{r},E')\ \phi\ (\underline{r},E',t)\ P\ (E' \to E)dE'. \qquad (1.2.3)$$

The production rate of fission neutrons is

$$d\underline{r}\ dE\ \nu X(E) \int_0^{10\ \text{Mev}} \Sigma_f(\underline{r},E')\ \phi\ (\underline{r},E',t). \qquad (1.2.4)$$

X(E)dE is the fraction of fission neutrons emitted in the range E to E + dE, ν is the number of fission neutrons produced per fission and $\Sigma_f(\underline{r},E')$ is the macroscopic fission cross section.

The rate of leakage is

$$-\ d\underline{r}\ dE\ \nabla \cdot D(\underline{r},E)\ \nabla\phi\ (\underline{r},E,t) \qquad (1.2.5)$$

and the rate at which neutrons are removed by all interactions is

$$d\underline{r}\ dE\ \Sigma_t(\underline{r},E)\ \phi\ (\underline{r},E,t), \qquad (1.2.6)$$

where $\Sigma_t(\underline{r},E)$ is the total macroscopic cross section.

The time rate of change of the number of neutrons in $d\underline{r}\ dE$ is

$$d\underline{r}\ dE\ \frac{1}{v}\frac{\partial\phi(\underline{r},E,t)}{\partial t}. \qquad (1.2.7)$$

Neutron conservation leads to an energy dependent diffusion equation for the neutron flux per unit energy, which is

$$\frac{1}{v}\frac{\partial\phi(\underline{r},E,t)}{\partial t} = \nabla\cdot D(\underline{r},E)\nabla\phi(\underline{r},E,t) - \Sigma_t(\underline{r},E)\phi(\underline{r},E,t) + \int_0^{10\ \text{Mev}} \Sigma_s(\underline{r},E')\phi(\underline{r},E',t)P(E'\to E)dE'$$

$$+ \nu X(E)\int_0^{10\ \text{Mev}} \Sigma_f(\underline{r},E')\phi(\underline{r},E',t)dE'. \qquad (1.2.7)$$

This is the basic form of the diffusion equation used in the development of the Reactor Statics and Reactor Dynamics Models. Scattering by more than one isotope merely gives an additional term having the same form as the in-scattering term in Equation (1.2.7) for each isotope. Similarly, fission of more than one isotope yields additional fission neutron production terms.

1.3 Steady State Epithermal Multi-Group Equations

In the epithermal energy region, above ~1 ev, the energy exchange with moderator can be calculated as though the moderator nuclei are at rest relative to neutrons. Also, the elastic scattering with low mass moderators is isotropic for energies up to the Mev region. The slight anisotropy at higher energies will be neglected. From elementary mechanics (cf., Glasstone and Sesonske, pp. 127-132), the probability that neutrons scattered with initial energy, E', have final energy in the range E to E + dE is

$$P(E'\to E)dE = \frac{dE}{(1-\alpha)E'} \text{ for } E \le E' \le \frac{E}{\alpha} \qquad (1.3.1)$$

$$= 0 \quad \text{otherwise}$$

where

$$\alpha = \left(\frac{A - 1}{A + 1}\right)^2$$

and

$$A = \frac{\text{Mass of scattering nucleus}}{\text{Mass of neutron}}.$$

For example, if the moderator is carbon, $\alpha = 0.716$ and neutrons can be scattered into dE at E only if their initial energies are in the range of E to 1.40 E. Note, also, that Equation (1.3.1) states that neutrons are uniformly scattered over the range E' to αE'.

The epithermal energy region can be divided into a number of contiguous energy groups. The g th group has the energy range, $E_g - E_{g-1}$, let g = 1 denote highest energy group, g = 2 the next highest etc. Generally, a reactor core and reflector or blanket can be divided into a relatively small number of regions, within each of which, the macroscopic cross sections are independent of $\underline{r}$. Thus, setting $\partial\phi/\partial t = 0$, using $P(E' \to E)$ given by Equation (1.3.1) and integrating Equation (1.2.7) with respect to E over the range, $E_g - E_{g-1}$, gives for each such reactor region,

$$\int_{E_{g-1}}^{E_g} D(E)\ \nabla^2\phi\ (\underline{r},E)\,dE - \int_{E_{g-1}}^{E_g} \Sigma_t(E)\ \phi\ (\underline{r},E)\,dE + \int_{E_{g-1}}^{E_g} dE \int_{E}^{E/\alpha} \Sigma_s(E')\ \phi\ (\underline{r},E')\ \frac{dE'}{(1-\alpha)E'}$$

$$+ \nu \int_{E_{g-1}}^{E_g} X(E)\,dE \int_{0}^{10\ \text{Mev}} \Sigma_f(E')\ \phi\ (\underline{r},E')\,dE' = 0. \quad (1.3.2)$$

Next, assume that $\phi(\underline{r},E)$ is separable over the range $E_g - E_{g-1}$ throughout the region; i.e., let

$$\phi(\underline{r},E) = G(\underline{R})\ F(E).$$

Then

$$\int_{E_{g-1}}^{E_g} D(E)\nabla^2\phi(\underline{r},E)\,dE = \nabla^2 G(\underline{r}) \int_{E_{g-1}}^{E_g} D(E)F(E)\,dE. \qquad (1.3.3)$$

Defining $D^{(g)}$ as the spectrum average value of D(E); i.e.,

$$D^{(g)} \equiv \frac{\int_{E_{g-1}}^{E_g} D(E)F(E)\,dE}{\int_{E_{g-1}}^{E_g} F(E)\,dE}, \qquad (1.3.4)$$

and noting that the group flux is

$$\phi_g(\underline{r}) = \int_{E_{g-1}}^{E_g} \phi(\underline{r},E)\,dE = G(\underline{r}) \int_{E_{g-1}}^{E_g} F(E)\,dE, \qquad (1.3.5)$$

the leakage term of the group equation becomes

$$\int_{E_{g-1}}^{E_g} D(E)\nabla^2\phi(\underline{r},E)\,dE = D^{(g)}\nabla^2\phi_g(\underline{r}). \qquad (1.3.6)$$

Similarly,

$$\int_{E_{g-1}}^{E_g} \Sigma_t(E)\phi(\underline{r},E)\,dE = \Sigma_t{}^{(g)}\phi_g(\underline{r}), \qquad (1.3.7)$$

with

$$\Sigma_t^{(g)} = \frac{\int_{E_{g-1}}^{E_g} \Sigma_t^{(g)}(E) F(E)\, dE}{\int_{E_{g-1}}^{E_g} F(E)\, dE} . \qquad (1.3.8)$$

Also,

$$\int_{E_{g-1}}^{E_g} dE \int_{E}^{E/\alpha} \Sigma_s(E')\, \phi\, (\underline{r},E')\, \frac{dE'}{(1-\alpha)E'} = \sum_{j=1}^{g} \Sigma_r(j \to g) \Phi_j(\underline{r}) . \qquad (1.3.9)$$

$\Sigma_r(j \to g)$ is the transfer cross section from group j to group g, which when multiplied by $\Phi_j(\underline{r})$ gives the rate at which neutrons are scattered from group j to group g per unit volume at $\underline{r}$. The transfer cross section is

$$\Sigma_r(j \to g) = \frac{\int_{E_{g-1}}^{E_g} dE \int_{E_{g-1}}^{E_g} \Sigma_s(E')F(E') \frac{dE'}{(1-\alpha)E'}}{\int_{E_{g-1}}^{E_g} F(E)\, dE} . \qquad (1.3.10)$$

In the transfer cross section, the upper limit of the integration over E' is E_g/α, since $P(E' \to E)$ vanishes for higher energies.

If inelastic scattering is important, Equation (1.2.7) will contain a term of the form,

$$\int_{E}^{10\ \text{Mev}} \Sigma_{in}(E')F(E')K(E' \to E)dE',$$

where $K(E' \rightarrow E)$ is the transfer probability for inelastic scattering and $\Sigma_{in}(E')$ is the total inelastic cross section for each inelastic scatterer. The inelastic transfer cross section is calculated in the same way as the elastic transfer cross section.

The fission neutron production term in Equation (1.3.2) becomes

$$\nu X^{(g)} \sum_{g=1}^{N} \Sigma_f^{(g)} \phi_g(\underline{r}).$$

The summation is over all groups including a thermal or slow neutron group. The $\Sigma_f^{(g)}$ is the average value of $\Sigma_f(E)$ over the spectrum $F(E)$ and

$$X^{(g)} = \int_{E_{g-1}}^{E_g} X(E)\,dE.$$

The epithermal multi-group equations, thus have the form,

$$D^{(g)} \nabla \phi_g(\underline{r}) - \Sigma_t^{(g)} \phi(\underline{r}) + \sum_{j=1}^{g} \Sigma_r(j \rightarrow g) \phi_j(\underline{r}) + \nu X^{(g)} \sum_{g=1}^{N} \Sigma_f^{(g)} \phi_g(\underline{r}) = 0. \quad (1.3.12)$$

$$g = 1, 2 \ldots N - 1.$$

N is the thermal or slow neutron group which is discussed in section 1.4.

Turning, now, to the calculation of the epithermal group constants, the simplest approximation is to use an infinite medium spectrum. Equation (1.2.7) reduces to

$$- \Sigma_t(E) F(E) + \int_{E}^{E/\alpha} \Sigma_s(E') F(E') \frac{dE'}{(1 - \alpha)E'} + X(E) = 0. \quad (1.3.13)$$

$F(E)$ is the neutron spectrum normalized to one fission neutron produced per unit volume and unit time; i.e.,

$$\nu \int_0^{10\ \text{Mev}} \Sigma_f(E')\phi(E')dE' = 1. \qquad (1.3.14)$$

The accuracy of this approximation for the flux spectrum within each group increases with decreasing energy width of the group. Thus, for a fairly large number of groups, say 20 to 30, F(E) can be obtained by numerical solution of Equation (1.3.13).

Another approximation which is widely used is based on asymptotic reactor theory. If the cross sections are independent of $\underline{r}$ over a region which is large compared to a group diffusion length, defined by,

$$L_g = \sqrt{\frac{D^{(g)}}{\Sigma_t^{(g)}}} \qquad (1.3.15)$$

the neutron flux energy distribution will assume an asymptotic distribution which is separable in space and energy over a region which is about L_g away from interfaces with other regions.

As before, $\phi(\underline{r},E) = G(\underline{r})\ F(E)$. Furthermore, G(R) will satisfy the Helmholtz equation,

$$\nabla^2 G(\underline{r}) + B^2 G(\underline{r}) = 0. \qquad (1.3.16)$$

The B^2 is the material buckling (cf., Glasstone and Sesonske, pp. 157-162). It is a function of the macroscopic cross sections and multiplying and scattering properties of the medium. If the infinite medium multiplication factor, k_∞, is greater than zero, B^2 will be positive. Conversely, if $k_\infty < 0$, B^2 will be negative.

Asymptotic reactor theory can be used for the calculation of the critical size of a bare geometrically simple homogeneous reactor. The spatial distribution of the flux per unit energy for all neutron energies satisfies the Helmholtz equation,

$$\nabla^2 G(\underline{r}) + B_g^2 G(\underline{r}) = 0, \qquad (1.3.17)$$

where B_g^2 is the geometric buckling which is determined by setting $G(\underline{r}) = 0$ on the extrapolated surface of the reactor. For example, a spherical reactor of extrapolated radius, R, would have the spatial distribution,

$$G(\underline{r}) = \frac{\sin \frac{\pi r}{R}}{r},$$

and

$$B_g = \frac{\pi}{R}.$$

If the material buckling is equal to the geometric buckling, the reactor is critical.

The asymptotic reactor theory approximation for computing the neutron spectrum yields the following equation for F(E),

$$-\left[D(E)B^2+\Sigma_t(E)\right] F(E) + \int_E^{E/\alpha} \Sigma_s(E')F(E')\frac{dE'}{(1-\alpha)E'} + X(E) = 0. \qquad (1.3.18)$$

The group constants are computed using this solution for F(E).

The choice of the buckling to use in Equation (1.3.18) is made by trial and error. A guess is made, the spectrum and multi-group constants calculated which are then used to calculate the spatial distribution of the group fluxes.

Inspection of the group fluxes will indicate whether or not the average leakage, $-D^{(g)}B^2\phi_g$, over the region is consistent with the value of B^2. If not, a different value of B^2 can be selected and the process repeated.

This approximation is the diffusion theory analog of the consistent P-1 and B-1 approximations used in reactor design codes[8].

In the neutron slowing down module, RS-3, resonance absorption is treated separately as discussed in Module RS-4. Neutron absorption is divided into a smooth component which is averaged over the calculated spectra and a resonance component. Resonance escape probabilities are computed for each group.

1.4 Slow Neutron Diffusion Equation

In the thermalization energy region below about 1 ev energy exchanges between neutrons and moderator are quite complicated as discussed in section 1.1. However, because of the strong energy coupling between the moderator and neutrons, the medium imposes a definite energy distribution on the neutrons which can be calculated by several more or less sophisticated models[9]. This distribution is used to calculate the slow or "thermal" group constants for a one group diffusion equation.

The steady state conservation equation is

$$L_s\nabla^2\phi_s(\underline{r},t) - \Sigma_{as}\phi_s(\underline{r},t) + q(\underline{r},t) = 0. \qquad (1.4.1)$$

The slow group flux is the integral of the distribution from zero to the maximum energy of the thermalization region, E_c; i.e.,

$$\phi_s(\underline{r},t) = \int_0^{E_c} \phi_s(\underline{r},E,t)\,dE \qquad (1.4.2)$$

$q(\underline{r},t)$ is the source of neutrons slowing down below E_c and is given by

$$q(\underline{r},E_c,t) = \int_{E_c}^{E_c/\alpha} \Sigma_s(E')\phi(\underline{r},E',t)\left[\frac{E_c - \alpha E'}{(1-\alpha)E'}\right] dE'. \qquad (1.4.3)$$

This equation for q can be derived simply by inspection. Neutrons at E' are scattered uniformly over the energy interval, $(1 - \alpha)E'$. The fraction that are scattered below E_c is $(E_c - \alpha E')/(1 - \alpha E')$.

The slow neutron diffusion coefficient and absorption cross section are average values weighted by the slow neutron flux energy distribution and ϕ_s is the total slow neutron flux.

The calculation of slow neutron spectra using the hydrogen gas model is discussed in Modules RS-5 and 6.

1.5 Structure and Organization of the Reactor Statics Modules

The following Reactor Statics Modules are:

RS Number	Title
2	One Group Neutron Diffusion Theory
3	Neutron Slowing Down and Epithermal Group Constants
4	Resonance Absorption
5	Slow Neutron Disadvantage Factors
6	Slow Neutron Spectra and Group Constants
7	Three Group Criticality Program
8	Multi-Group Constants for Fast Reactors

RS-2 introduces the student to the concept of the critical eigenvalue, k, the discretization of a one group diffusion equation and the numerical solution of the one group reactor model for k and the spatial neutron flux distribution.

For slow neutron reactor calculations, the neutron energy range is divided into three groups, fast, resonance and slow. The division follows naturally from the different neutron-nuclei interactions in each energy group. Thus, all fission neutrons are produced with energies in the fast group. Also, neutrons cannot be removed from the fast group by inelastic scattering. The resonance group contains all resolved resonances and is sufficiently wide so that neutron transfer by elastic scattering from the fast group to the slow group can be neglected. Finally, the entire thermalization process is confined to the slow group. Their energy range and lethargy widths are given in Table 1.5.1. They are called macro-groups to distinguish them from the micro-groups used in neutron spectrum calculations.

Table 1.5.1

Slow Neutron Reactor Macro-Groups

Macro-Group	Energy Range (ev)	Lethargy Width
Fast	$5.531 \times 10^3 - 1.000 \times 10^7$	7.50
Resonance	$0.640 - 5.531 \times 10^3$	9.06
Slow	$0.000 - 0.64$	--

The group constants can be calculated for each reactor region using RS-3 and 4 for the fast and resonance group and RS-5 and 6 for the slow group. One dimensional criticality and neutron distribution problems can be solved using

these group constants as input into the three group criticality program presented in RS-8.

The last module includes a separate program for the calculations of neutron spectra and group constants for use in fast breeder reactor calculations. It can be used with the criticality programs in RS-2 and RS-8.

A typical criticality and power distribution calculation for a low enriched uranium oxide slow neutron reactor using the three group model can be made using the following procedure. First, the reactor is divided into regions, within which material properties are constant. For each region, the isotopic number densities in the fuel, clad and moderator are calculated. The fuel lattice cell geometry and the average temperature of the fuel is calculated using the Thermal-Hydraulics Module, TH-1. This provides input data for each reactor region for the following computer programs;

Module	Program Name	Output
RS-3	FARCON	Fast and Resonance Group Constants
RS-6	SLOWCON	Slow Group Constants

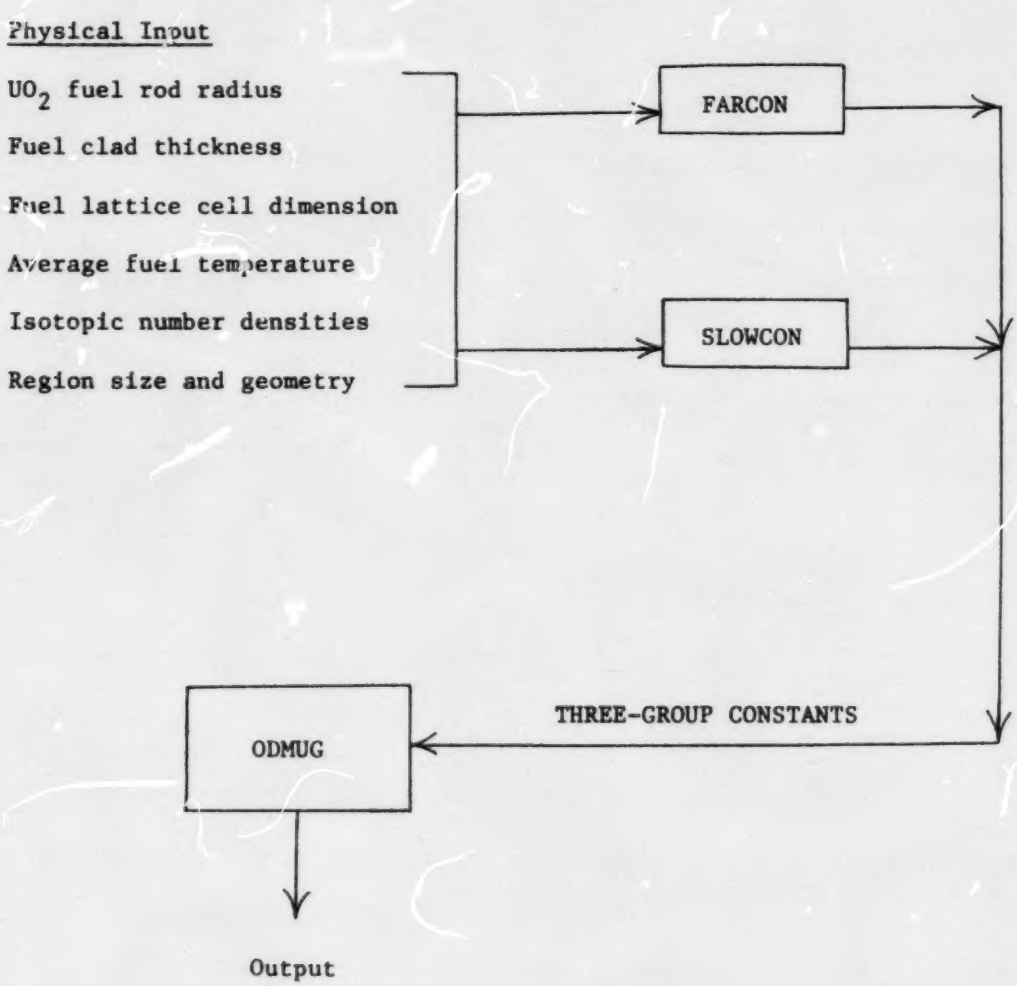
This output information is used as input data to the three group-one dimensional code, ODMUG, presented in RS-7. ODMUG can be used to compute the critical eigenvalue, k, and the uniform slow neutron poison cross section required to make the reactor critical. The output of ODMUG includes;

1) Critical eigenvalue of "unpoisoned" reactor,
2) Poison cross section needed for criticality,
3) Spatial flux distribution,

4) Spatial power distribution and

5) Total power generated and power density in each region.

The sequence of calculations is shown in the following diagram.



Fast reactor calculations can be made by using FASTCON to generate group constants for either ODOG or ODMUG.

References

1. Smyth, Henry DeWolf, Atomic Energy for Military Purposes, Princeton University Press, Princeton, N.J. (1945).

2. Smyth, Henry DeWolf, Ibid.

3. Glasstone, S. and Edlund, M. C., The Elements of Nuclear Reactor Theory, D. Van Nostrand Company, Inc., Princeton, N.J. (1952).

4. Weinberg, A. M. and Wigner, E. P., The Physical Theory of Neutron Chain Reactors, The University of Chicago Press, Chicago, Ill. (1958).

5. Galanin, A. D., Thermal Reactor Theory, Pergamon Press, New York, N.Y. (1960).

6. Lamarsh, J. R., Introduction to Nuclear Reactor Theory, Addison-Wesley Publishing Company, Inc., Reading, Mass. (1966).

7. Bell, G. I. and Glasstone, S., Nuclear Reactor Theory, Van Nostrand Reinhold Company, New York, N.Y. (1970).

8. Bell, G. I. and Glasstone, S., Ibid.